SOLUTION OF A PROBLEM.

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Given three circles touching each other externally; if another group of three be drawn touching each other in like manner, and so situated that each circle of either group touches two of the other, to prove that the square of the common tangent of any two of the circles, which do not touch each other, is equal to twice the product of their diameters.

Solution. This property is easily proved by means of the following principle, due to Mr. Casey, and found on page 113, Salmon's Conics, Fourth Edition: "If four circles be all touched by the same fifth circle, the lengths of their common tangents are connected by the following relation: $\overline{12}\,.\,\overline{34} \pm \overline{14}\,.\,\overline{23} \pm \overline{13}\,.\,\overline{24} = 0$, where $\overline{12}$ denotes the length of a common tangent to the first and second circles, &c."

Denoting the circles of the first group and their radii by a, b, c, and of the second by x, y, z, there will be three pairs of non-touching circles, viz; a,x, b,y, and $c;z.$, Let T, T', T'' be their common tangents, then as z is touched by a, b, x, y, which touch each other consecutively,

$$2\sqrt{(ab)}\times2\sqrt{(xy)} + 2\sqrt{(ay)}\times2\sqrt{(bx)} = T.T',$$

or $$8\sqrt{(abxy)} = T.T'.$$

Similarly $$8\sqrt{(bcyz)} = T'.T'',$$

and $$8\sqrt{(cazx)} = T''.T,$$ whence, by elimination,

$$T^2 = 8ax = \text{twice the product of the diameters of } a \text{ and } x.$$

NOTE.—TO THE STUDENT OF PHYSICAL ASTRONOMY: When you undartake the study of the theory of Perturbations of the planetary motions, do not take up any extended work on the subject and attempt to master all its details at one reading, but try to get the *general principles* of the subject — not the most general methods — and as soon as you have done that, commence to prepare with your own hands a treatise on the subject for your own use. Continue to *think* on the subject till you can explain to another all the details so far as you go, and do this in your treatise. By all means commence with the *Lunar Theory,* and I should say the "*Variation of Constants*".

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